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(Write Roll Number from left side exactly as in the Admit Card)

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1. _____
2. _____

Question Booklet Series

Y

Question Booklet No.

(Identical with OMR Answer Sheet Number)

1618

PAPER-II

Subject Code : 16

PHYSICAL SCIENCES

Time : 2 Hours

Maximum Marks: 200

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
 - (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
 - (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
 - (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
 - (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D). You have to darken the circle as indicated below on the correct response against each question.

Example: (A) (B) (C) (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. Use only Black Ball point pen.
11. Use of any calculator or mobile phone etc. is strictly prohibited.
12. There are no negative marks for incorrect answers.

[Please Turn Over]

PHYSICAL SCIENCES

PAPER II

1. $|\psi\rangle$ is an eigenvector of an operator \hat{A} . The operator $\hat{B} = \hat{U}\hat{A}\hat{U}^+$ where \hat{U} is a unitary operator. Which one of the following is an eigenvector of the operator $e^{\hat{B}}$?

- (A) $|\psi\rangle$
- (B) $\hat{U}|\psi\rangle$
- (C) $\hat{U}^+|\psi\rangle$
- (D) $e^{\hat{U}}|\psi\rangle$

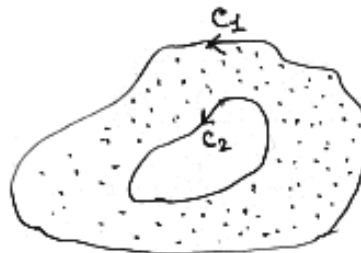
2. Electric field in a region R , is given by $\vec{E} = y^2 \hat{i} + x^2 \hat{j}$, then

- (A) the region does not have any magnetic field.
- (B) x and y components of the magnetic field are time independent.
- (C) charge density is increasing function of time.
- (D) charge density is decreasing function of time.

3. Galileo dropped two unequal masses simultaneously from the top of the leaning tower of Pisa and found that they hit the ground simultaneously. This proved that

- (A) acceleration is inversely proportional to mass.
- (B) gravitational force is independent of mass.
- (C) falling bodies eventually reach the same terminal velocity.
- (D) gravitational mass is equal to inertial mass.

4. If $f(z)$ is analytic within and on the boundary of a region bounded by two closed curves C_1 and C_2 then



- (A) $\oint_{C_1} f(z)dz$ is greater than $\oint_{C_2} f(z)dz$
- (B) $\oint_{C_1} f(z)dz$ is less than $\oint_{C_2} f(z)dz$
- (C) $\oint_{C_1} f(z)dz$ is equal to $\oint_{C_2} f(z)dz$
- (D) None of the above

5. The dispersion relation for a simple cubic crystal is given by $E(\vec{k}) = E_0 - 2\beta(\cos k_x a + \cos k_y a + \cos k_z a)$, where β is a constant having the dimension of energy. The width of the energy band is

- (A) 2β
- (B) 4β
- (C) 8β
- (D) 12β

6. The Fermi energy of a free electron gas is 4.5 eV. The probability of finding an electron with “up-spin” ($S_z = +1$) and energy 4.5 eV at $T = 0$ is

- (A) 1
- (B) 0.5
- (C) 0.25
- (D) nearly zero

7. Relative intensities of successive lines in the obtained Raman spectra of nitrogen molecule is

- (A) 3 : 4
- (B) 1 : 2
- (C) 2 : 3
- (D) 1 : 3

8. For a 100% amplitude modulated (AM) signal the ratio of carrier power to total AM signal power at the output is

- (A) 2 : 1
- (B) 5 : 2
- (C) 2 : 3
- (D) 1 : 2

9. To design a transistorized phase shift oscillator with 3 identical R-C sections in its feedback network, the transistor h_{fe} parameter

- (A) can have any value.
- (B) can have a value in the range 20–35.8.
- (C) should have a value exceeding 44.6.
- (D) must have a value exceeding 100.

10. A proton is accelerated to an energy E and shot directly at a nucleus of nitrogen (${}^{14}_7\text{N}$). In order for the proton to overcome the Coulomb repulsion and reach the nucleus, its energy must be at least

- (A) 250 keV
- (B) 45 MeV
- (C) 105 MeV
- (D) 1.8 GeV

11. A Hamiltonian is described by the canonical coordinate q and canonical momentum p . A new coordinate Q is defined as $Q(t) = q(t + \tau) + p(t + \tau)$, where t is the time and τ is a constant. The new canonical momentum $P(t)$ can be expressed as

- (A) $p(t + \tau) - q(t + \tau)$
- (B) $p(t + \tau) - q(t - \tau)$
- (C) $\frac{1}{2}[p(t - \tau) - q(t + \tau)]$
- (D) $\frac{1}{2}[p(t + \tau) - q(t + \tau)]$

12. Consider the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Its eigenvalues and

normalized eigenvectors are

- (A) $\left[0, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right], \left[1, \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right], \left[2, \frac{1}{2}\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}\right]$
- (B) $\left[0, \frac{1}{2}\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}\right], \left[0, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right], \left[3, \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right]$
- (C) $\left[1, \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right], \left[1, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right], \left[1, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right]$
- (D) $\left[1, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right], \left[1, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right], \left[1, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right]$

13. If ψ_1, ψ_2, ψ_3 are three normalized energy eigenstates of a system corresponding to the energy ϵ_1, ϵ_2 and ϵ_3 respectively and a state ϕ is represented as $\phi = 3\psi_1 + 2\psi_2 + \psi_3$, then

- (A) a single measurement of energy will yield a value $3\epsilon_1 + 2\epsilon_2 + \epsilon_3$
- (B) the average value of energy measured on a large number of replica of this system in the state ϕ will yield a value $3\epsilon_1 + 2\epsilon_2 + \epsilon_3$
- (C) a single measurement will yield a value ϵ_3 with a probability $\frac{1}{6}$
- (D) a single measurement will yield a value ϵ_2 with a probability $\frac{2}{7}$

14. A particle starts out in a linear combination ' Ψ ' of two stationary states ψ_1 and ψ_2 corresponding to energy eigenvalues E_1 and E_2 respectively, namely, $\Psi(x,0) = \psi_1(x) + \psi_2(x)$. If $E_1 \neq E_2$, at any latter time 't', the probability density will

- (A) be independent of time
- (B) vary inversely with time
- (C) be sinusoidally oscillating with time
- (D) be a linear function of time

15. A polarized dielectric sphere of radius R with polarization $\vec{p}(\vec{r}) = k\vec{r}$ is placed at the origin. The electric field inside and outside the sphere are

- (A) $-\frac{k\vec{r}}{\epsilon_0}, 0$
- (B) $-\frac{R^3 k\vec{r}}{4\pi\epsilon_0 r^3}, 0$
- (C) $0, \frac{R^3 k\vec{r}}{4\pi\epsilon_0 r^3}$
- (D) $-\frac{k\vec{r}}{\epsilon_0}, \frac{k\vec{r}}{\epsilon_0}$

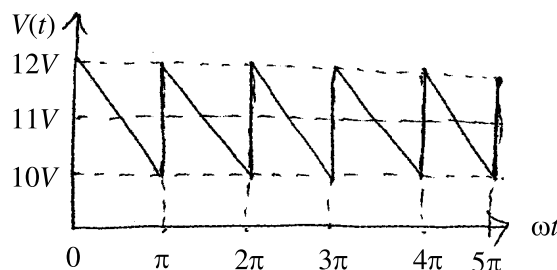
16. In a microcanonical ensemble, a system A of fixed volume is in contact with a large reservoir B. Then

- (A) A can exchange only entropy with B.
- (B) A can exchange only particles with B.
- (C) A can exchange neither energy nor particles with B.
- (D) A can exchange both energy and particles with B.

17. Due to fine structure each of the line in Balmer series is split into

- (A) 2 lines
- (B) 3 lines
- (C) 5 lines
- (D) 7 lines

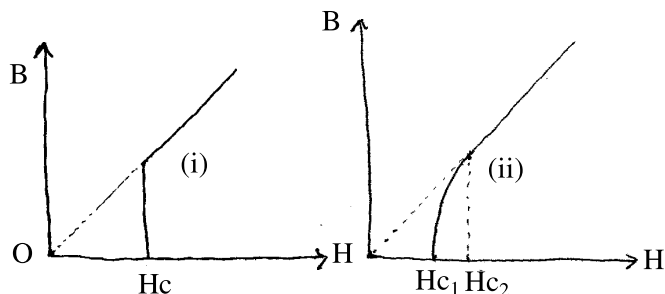
18. The following saw-tooth ac voltage, $v(t)$, has a *rms* value of the ac components equal to



- (A) $\sqrt{2}V$
- (B) $\frac{1}{\sqrt{2}}V$
- (C) $\sqrt{3}V$
- (D) $\frac{1}{\sqrt{3}}V$

19. The figures (i) and (ii) below show the flux penetration as a function of an external magnetic field for

- (A) a normal metal (i) and a superconductor (ii)
- (B) a normal metal (ii) and a superconductor (i)
- (C) a type-I superconductor (i) and a type-II superconductor (ii)
- (D) a normal metal, both in fig. (i) and in fig. (ii)



20. For switching operation, the BJT should be operated in the

- (A) Active region only
- (B) Saturation region only
- (C) Cut-off region only
- (D) Cut-off and saturation region

21. A block sits on a plane that is inclined at an angle. Assume that the friction force is large enough to keep the block at rest. For what value of the angle is the horizontal component of the friction force maximum?

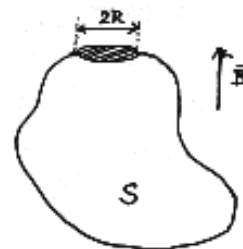
- (A) π
- (B) $\frac{\pi}{2}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{4}$

22. Let \vec{a} and \vec{b} be two distinct three dimensional vectors. Then the component of \vec{b} that is perpendicular to \vec{a} is given by

- (A) $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{a^2}$
- (B) $\frac{(\vec{a} \cdot \vec{b})\vec{b}}{b^2}$
- (C) $\frac{\vec{b} \times (\vec{a} \times \vec{b})}{b^2}$
- (D) $\frac{(\vec{b} \cdot \vec{a})\vec{a}}{a^2}$

23. Consider a surface S , as shown in the figure, which has a circular hole of radius R at one end. If this is placed in a uniform magnetic field \vec{B} , as shown in the figure, the flux of \vec{B} through the surface S is

- (A) $-4\pi R^2 |\vec{B}|$
- (B) $-\pi R^2 |\vec{B}|$
- (C) $\pi R^2 |\vec{B}|$
- (D) $4\pi R^2 |\vec{B}|$



24. Suppose ΔE_e , ΔE_v and ΔE_r are the energy separations between two neighbouring states in the electronic, vibrational and rotational spectra of a molecule respectively. Then which one of the following is correct?

- (A) $\Delta E_e \gg \Delta E_v \gg \Delta E_r$
- (B) $\Delta E_e = \Delta E_v \gg \Delta E_r$
- (C) $\Delta E_e = \Delta E_r \gg \Delta E_v$
- (D) $\Delta E_e \ll \Delta E_v \ll \Delta E_r$

25. Semiconductor particle detectors consist of an array of

- (A) silicon crystals cooled in liquid nitrogen.
- (B) p-n junctions in reverse bias.
- (C) nano-sized MOSFETs sandwiched between gold foil.
- (D) Josephson junctions cooled in liquid helium.

26. A large number of 100Ω resistors is measured with mean value of 100.00Ω and standard deviation of 0.20Ω . If the data is dispersed normally what percentage of all the resistors on the average will have values that lie between the limits $\pm 0.20 \Omega$?

- (A) 50%
- (B) 68%
- (C) 80%
- (D) 95%

27. In the powder diffraction pattern of an fcc sample, the first peak arises at 30° . The second peak will appear at

- (A) $\sin^{-1} \frac{1}{\sqrt{3}}$
- (B) $\sin^{-1} \frac{2}{\sqrt{3}}$
- (C) $\sin^{-1} \frac{1}{2\sqrt{3}}$
- (D) $\sin^{-1} \frac{1}{\sqrt{2}}$

28. Planck's formula for black body radiation reduces to Rayleigh-Jeans and Wien's formula for

- (A) small and large wavelength
- (B) large and small wavelength
- (C) large and complex wavelength
- (D) complex and large wavelength, respectively

29. The power of the radiation emitted from an oscillating dipole, $\vec{p}(t) = \vec{p}_0 \cos \omega t$ is

- (A) isotropic
- (B) maximum along \vec{p}
- (C) maximum along a perpendicular direction of \vec{p}
- (D) maximum along a direction 45° to \vec{p}

30. Identify the correct statement about the following infinite series:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \dots$$

- (A) The series is not convergent.
- (B) The series is absolutely convergent.
- (C) The series is convergent but not absolutely convergent.
- (D) None of the above statement is true.

31. What is the proper time interval between the occurrence of two events if in one inertial frame events are separated by 7.5×10^8 m and occur 6.5 sec. apart?

- (A) 6.5 sec
- (B) 6.00 sec
- (C) 5.75 sec
- (D) 5.00 sec

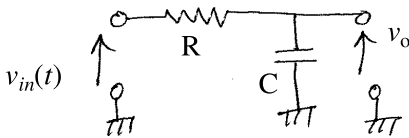
32. The black body temperature of 6000 K emits a radiation whose intensity spectrum peaks at 600 nm. If the temperature is reduced to 300 K, the spectrum will peak at

- (A) 120 μm
- (B) 12 μm
- (C) 12 mm
- (D) 120 mm

33. Consider a nucleus with spin $\frac{1}{2}$. If this has an intrinsic magnetic moment P_n and an electron has intrinsic magnetic moment P_e , the ratio P_n/P_e will have magnitude

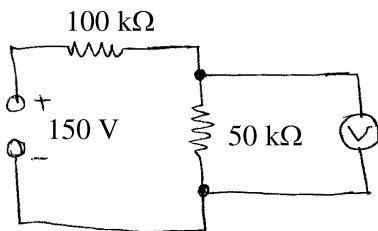
- (A) zero, because the nucleus behaves like a point.
- (B) $\gg 1$, because the nucleus is much smaller than the electron.
- (C) $\ll 1$, because the nucleus is much heavier than the electron.
- (D) $\ll 1$, because the strong interaction is much stronger than the electromagnetic interaction.

34. A sinusoidal signal voltage, $v_{in}(t) = V\sin\omega t$ is applied to the input of the following circuit with $RC = 0.1$ sec. The output voltage, v_o , will



- (A) lead the input voltage by an angle $\phi = \tan^{-1}\left(\frac{\omega}{10}\right)$
 (B) lead the input voltage by an angle $\phi = \tan^{-1}(10\omega)$
 (C) lag behind the input voltage by an angle $\phi = \tan^{-1}\left(\frac{\omega}{10}\right)$
 (D) lag behind the input voltage by an angle $\phi = \frac{\pi}{2}$ radian

35. The voltage across the 50-k Ω resistor in the circuit is measured using a voltmeter with sensitivity of 1000 Ω/V and range of 0–50 V. The error of measurement expressed as a percentage of the true value will be



- (A) 4%
 (B) 5%
 (C) 40%
 (D) 50%

36. A diatomic linear structure with masses M_1 and M_2 ($M_1 > M_2$) connected by the force constant C has lattice constant a . The angular frequency of the optical branch at the zone boundary:

- (A) $\left(\frac{2C}{M_2}\right)^{1/2}$
 (B) $\left(\frac{2C}{M_1}\right)^{1/2}$
 (C) $\left(\frac{C}{M_2}\right)^{1/2}$
 (D) $\left(\frac{C}{M_1}\right)^{1/2}$

37. An ensemble of N three level systems with energies $\epsilon = -\epsilon_0, 0, \epsilon_0$ is in thermal equilibrium at temperature T . If $\frac{\epsilon_0}{K_B T} = 2$, the probability of finding the system in the level $\epsilon = 0$ is

- (A) $\cosh(2)$
 (B) $(\cosh(2))^{-1}$
 (C) $(2 \cosh(2))^{-1}$
 (D) $(1 + 2 \cosh(2))^{-1}$

38. \hat{A} and \hat{B} are two operators defined by $\hat{A}\psi(x) = \psi(x) + x$ and $\hat{B}\psi(x) = \frac{d\psi(x)}{dx} + 2\psi(x)$. Which one of the following statements is correct?

- (A) Both \hat{A} and \hat{B} are linear operators.
 (B) Both \hat{A} and \hat{B} are non-linear operators.
 (C) \hat{B} is a linear operator, but \hat{A} is not.
 (D) \hat{A} is a linear operator, but \hat{B} is not.

39. An operator \hat{A} is represented in certain orthonormal basis $\{|\psi_i\rangle, i=1, 2\}$ by the matrix $\hat{A} = \begin{pmatrix} 1 & \lambda \\ \lambda & 1 \end{pmatrix}$. If one constructs a 'new basis' by the kets $|\phi_1\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$ and $|\phi_2\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle - |\psi_2\rangle)$, then the same operator is represented in the new basis by

- (A) $\begin{pmatrix} \lambda & 1 \\ 1 & \lambda \end{pmatrix}$
 (B) $\begin{pmatrix} 1+\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}$
 (C) $\begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix}$
 (D) $\begin{pmatrix} \lambda & \lambda \\ 1 & -1 \end{pmatrix}$

40. The Fourier transform of the derivative of the Dirac δ -function, namely $\delta'(x)$, is proportional to

- (A) 0
 (B) 1
 (C) $\sin k$
 (D) ik

41. The Lagrangian of a free particle in spherical polar co-ordinate is $L = \frac{1}{2} [m\dot{r}^2 + mr^2\dot{\theta}^2 + mr^2\dot{\phi}^2 \sin^2 \theta]$.

The entity that is conserved is

- (A) $\frac{\partial L}{\partial \dot{r}}$
 (B) $\frac{\partial L}{\partial \dot{\theta}}$
 (C) $\frac{\partial L}{\partial \dot{\phi}}$
 (D) $\frac{\partial L}{\partial \dot{\phi}} + \dot{r}\dot{\theta}$

42. The Hamiltonian of a relativistic particle of rest mass m and momentum p is given by $H = \sqrt{p^2 + m^2} + V(x)$ in units in which the speed of light $c = 1$. The corresponding Lagrangian is

- (A) $L = m\sqrt{1 + \dot{x}^2} - V(x)$
 (B) $L = -m\sqrt{1 - \dot{x}^2} - V(x)$
 (C) $L = \sqrt{1 + m\dot{x}^2} - V(x)$
 (D) $L = \frac{1}{2} m\dot{x}^2 - V(x)$

43. A charge q is located at the centre of a cube of side a . Electric flux through any face of the cube is

- (A) $\frac{q}{\epsilon_0}$
 (B) $\frac{q}{4\pi\epsilon_0}$
 (C) $\frac{q}{2\epsilon_0}$
 (D) $\frac{q}{6\epsilon_0}$

44. The intrinsic carrier concentration of Ge at room temperature (300 K) is $2.4 \times 10^{13}/\text{c.c.}$. It is doped by acceptor impurity with a concentration of $10^{17}/\text{c.c.}$. The electron concentration in the doped Ge sample will be

- (A) $2.4 \times 10^{13}/\text{c.c.}$
 (B) $10^{17}/\text{c.c.}$
 (C) $4.16 \times 10^3/\text{c.c.}$
 (D) $5.76 \times 10^9/\text{c.c.}$

45. How does the specific heat of a Bose gas vary at very low temperature?

- (A) $C_v \propto T$
 (B) $C_v \propto T^{2/3}$
 (C) $C_v \propto T^{3/2}$
 (D) $C_v \propto T^{1/2}$

46. The meson K^0 , with strangeness quantum number -1 , decays through the modes $K^0 \rightarrow \pi^+ \pi^-$, $K^0 \rightarrow \pi^0 \pi^0$. These decays are due to

- (A) gravitational interactions
- (B) strong interactions
- (C) weak interactions
- (D) electromagnetic interactions

47. Any $0 \rightarrow 0$ gamma transition is forbidden due to

- (A) parity conservation
- (B) isospin asymmetry
- (C) angular momentum conservation
- (D) strong interactions

48. The temperature of a cavity of fixed volume is doubled. Which one of the following is true for black body radiation?

- (A) Its energy and number of photons both increases by 8 times.
- (B) Its energy increases by 8 times and the number of photons increases by 16 times.
- (C) Its energy increases 16 times and the number of photons increases by 8 times.
- (D) Its energy and number of photons both increase by 16 times.

49. The energy eigenvalue $E_n(\vec{k})$ in the n th band of an infinite periodic crystal is related to that for a wavevector $-\vec{k}$ by the relation

- (A) $E_n(\vec{k}) = -E_n(-\vec{k})$
- (B) $E_n(\vec{k}) = E_n(-\vec{k})$
- (C) $E_n(\vec{k}) = \sqrt{E_n(-\vec{k})}$
- (D) $E_n(\vec{k}) + E_n(-\vec{k}) = 2$

50. Consider the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

with boundary conditions at $t = 0$, $x = 1$, $\frac{dx}{dt} = 0$. The value of x at $t = 1$ is

- (A) $\frac{1}{e}$
- (B) $\frac{2}{e}$
- (C) e
- (D) 1

51. The eigenfunctions of the position operator (in one dimension) are of the form of a

- (A) Step function
- (B) Dirac delta function
- (C) Sine function
- (D) Exponentially decaying function

52. Which one of the following relations is *true* for chemical potential μ for the physical system (N, V, E)?

- (A) $\mu = H/N$
- (B) $\mu = G/N$
- (C) $\mu = A/N$
- (D) $\mu = E/N$

53. For designing a laser diode it is necessary to have a

- (A) direct band gap semiconductor.
- (B) indirect band gap semiconductor.
- (C) any kind of semiconductor direct or indirect.
- (D) Si or Ge as the semiconductor material.

54. The truth table below represents which of the following boolean expression?

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

- (A) $A + BC$
 (B) $\bar{A}C + B$
 (C) $\bar{A}C + AB$
 (D) $\bar{A}BC$

55. A parallel plate capacitor C with plates separated by d is connected to a resistance R. The displacement current on charging the capacitor with source V is

- (A) $\frac{V}{CRd} e^{-t/CR}$
 (B) $\frac{\epsilon_0 Vd}{CR} e^{-t/CR}$
 (C) $\frac{CRV}{\epsilon_0 d} e^{-t/CR}$
 (D) $\frac{\epsilon_0 V}{CRd}$

56. Consider circular orbits in a central force potential $V(r) = -\frac{k}{r^n}$, when $k > 0$ and $0 < n < 2$. If the time period of a circular orbit of radius R is T_1 and that of radius 2R is T_2 , then $\left(\frac{T_2}{T_1}\right)$

- (A) $2^{\frac{n}{2}}$
 (B) $2^{\frac{2}{3}n}$
 (C) $2^{\frac{n}{2}+1}$
 (D) 2^n

57. A (3×3) matrix M has $\text{Tr}[M] = 6$ and $\text{Tr}[M^2] = 26$. Which one of the following can be a possible set of eigenvalues of M?

- (A) $\{1, 1, 4\}$
 (B) $\{-1, 0, 7\}$
 (C) $\{-1, 3, 4\}$
 (D) $\{2, 2, 2\}$

58. Which one of the following wavefunction can be a bound state solution of Schrödinger equation for all values of x ?

- (A) $\sec x$
 (B) x^2
 (C) $e^{-|x|}$
 (D) $\log x$

59. The work done W during an isothermal process in which a gas expands from volume V_i to volume V_f is given by

- (A) $R(V_f - V_i) \log_e \frac{T_1}{T_2}$
 (B) $R(T_2 - T_1) \log_e \frac{V_f}{V_i}$
 (C) $RT \log_e \frac{V_f}{V_i}$
 (D) $RT \log_e \frac{V_i}{V_f}$

60. An electron, with a wavevector \vec{k} is incident on a square lattice of lattice constant 'a'. If $k_x = k_y = \pm \frac{\pi}{a}$, the electron will be "Bragg reflected" by

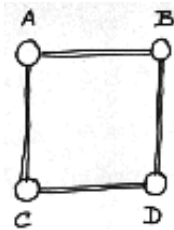
- (A) lattice planes parallel to the x -axis only.
- (B) lattice planes parallel to the y -axis only.
- (C) lattice planes parallel to both x and y -axes.
- (D) an infinite number of lattice planes aligned in any direction.

61. The state of polarization of an electromagnetic wave given by

$$\vec{E}(z, t) = \hat{i} E_0 \cos(kz - \omega t) - \hat{j} E_0 \cos(kz - \omega t) \text{ is}$$

- (A) linearly polarized
- (B) plane polarized
- (C) circularly polarized
- (D) unpolarized

62. Four massive balls A, B, C and D are connected by massless rigid rods, pivoted at the points A, B, C, D to rotate freely in all three dimensions. The number of degrees of freedom of this system is



- (A) 4
- (B) 8
- (C) 10
- (D) 12

63. The locus represented by the equation $|z - 3| + |z + 3| = 10$ is

- (A) Circle
- (B) Ellipse
- (C) Parabola
- (D) Hyperbola

64. The unit vector normal to the surface $3x^2 + 4y = z$ at the point (1, 1, 7) is

- (A) $\frac{1}{\sqrt{24}}(3\hat{i} + 4\hat{j} - \hat{k})$
- (B) $\frac{1}{\sqrt{51}}(6\hat{i} + 4\hat{j} - \hat{k})$
- (C) $\frac{1}{\sqrt{53}}(6\hat{i} + 4\hat{j} - \hat{k})$
- (D) $\frac{1}{\sqrt{51}}(\hat{i} + \hat{j} + 7\hat{k})$

65. If the force on some particle is given by $\vec{F} = -\hat{i}y + \hat{j}x$, then the work done by this force in moving the particle one complete turn along the closed curve $9x^2 + 4y^2 = 1$ is given by

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{3}$
- (C) $\frac{\pi}{48}$
- (D) $\frac{\pi}{2}$

66. The position (x, y, z) and momentum (p_x, p_y, p_z) coordinates satisfy the standard Poisson bracket relations. What is the value of the Poisson bracket $\{xy^2, p_z p_x\}$

- (A) $x p_z$
- (B) y^2
- (C) $y^2 p_x$
- (D) $y^2 p_z$

67. An infinite conducting plane (say, $x - y$ plane) is maintained at zero potential everywhere except within a circular region of radius r where the potential is V_0 . The electric field along the z -axis is

- (A) $\frac{V_0 z}{r+z}$
 (B) $\frac{V_0 r^2}{z^3}$
 (C) $\frac{V_0 r^2}{(r^2 + z^2)^{3/2}}$
 (D) $\frac{V_0}{z}$

68. Results of superposition of two sets of electric field

Set I: $E_x = E_1 \cos(\omega t - kz)$; $E_y = 0$

Set II: $E_x = E_3 \cos(\omega t - kz)$; $E_y = E_4 \cos(\omega t - kz - \pi/2)$
 will result

- (A) Plane polarized wave
 (B) Circularly polarized wave
 (C) Elliptically polarized wave
 (D) Unpolarized polarized wave

69. Spectral line width of the He-Ne LASER is 0.01 nm and the cross-sectional area of the beam is 0.01 cm². If the output power is 1 mW, the radiation intensity per unit wavelength is

- (A) 10¹⁰ W/cm³
 (B) 10⁸ W/cm³
 (C) 10⁻⁸ W/cm³
 (D) 10⁻¹⁰ W/cm³

70. One kilogram of ice melts at 0°C into water at the same temperature. The change in entropy (cal/K) is

- (A) ∞
 (B) 0
 (C) 273
 (D) 293

71. A junction field effect transistor (JFET) is a

- (A) Bipolar device
 (B) Unipolar device
 (C) Low input impedance device
 (D) High recombination noise device

72. The minimum number of two input NAND gates required to realize an EX-OR operation is

- (A) 2
 (B) 5
 (C) 3
 (D) 4

73. If 's' and 'p' represent hydrogen-like orbitals, then which one of the following statements is true?

- (A) Both $\langle 2s | x | 2p_y \rangle$ and $\langle 2p_x | x | 2p_y \rangle$ are non-zero.
 (B) $\langle 2p_x | x | 2p_y \rangle$ is non-zero, and $\langle 2s | x | 2p_y \rangle$ is zero.
 (C) $\langle 2s | x | 2p_y \rangle$ is non-zero and $\langle 2p_x | x | 2p_y \rangle$ is zero.
 (D) Both $\langle 2s | x | 2p_y \rangle$ and $\langle 2p_x | x | 2p_y \rangle$ are zero.

74. Which one of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$?

- (A) $(3 + 2x - iy)^7$
 (B) $(1 + x + iy)^4 (7 - x - iy)^3$
 (C) $(1 - x - iy)^4 (7 - x + iy)^3$
 (D) $(x + iy - 1)^{1/2}$

75. For a one-dimensional harmonic oscillator, the raising (a^+) and lowering (a) operators satisfy $[a, a^+] = 1$. Which one of the results below is correct?

- (A) $[a^n, (a^+)^n] = 1$
- (B) $[a^n, a^+] = 0$
- (C) $[a^+, a^n] = na^{n-1}$
- (D) $[a^n, a^+] = na^{n-1}$

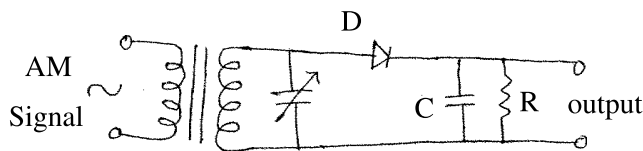
76. For a linear dielectric material which is isotropic and homogeneous, the relation between free charge density and bound charge density can be expressed as

- (A) $\rho_f = \frac{\epsilon_r \rho_b}{1 - \epsilon_r}$
- (B) $\rho_f = -\frac{\epsilon_0 \rho_b}{1 - \epsilon_0 \epsilon_r}$
- (C) $\rho_b = \frac{\rho_f}{1 - \epsilon_r}$
- (D) $\rho_b = -\frac{(1 - \epsilon_r)\rho_f}{\epsilon_0 \epsilon_r}$

77. An ensemble for which the density function $\rho(p, q, T) \propto \exp(-H(p, q)/kT)$, where $H(p, q)$ is the Hamiltonian for that ensemble is

- (A) micro canonical
- (B) canonical
- (C) grand canonical
- (D) macro canonical

78. The following circuit is that of a



- (A) Amplitude modulator
- (B) Balanced modulator
- (C) AM signal amplifier
- (D) Envelope detector

79. The current passing through a resistor of $100 \pm 0.2 \Omega$ is 2.00 ± 0.01 A. Maximum error in computing power dissipation will be

- (A) 1.2%
- (B) 0.8%
- (C) 0.5%
- (D) 0.2%

80. The hexadecimal equivalent of the octal number 225_8 is

- (A) 83
- (B) 95
- (C) 53
- (D) 59

81. The energy of an electron which is bound inside a nucleus of size around 1 fm is around

- (A) 1.2 eV
- (B) 1.2 keV
- (C) 1.2 MeV
- (D) 1.2 GeV

82. A particle moving in one dimension is subjected to a Lagrangian of the form $L = \frac{1}{2} \dot{x} \ln \dot{x} - \frac{1}{2} \omega^2 x^2$. The Hamiltonian H of this system will be

- (A) $H = \frac{1}{2} \exp(2p-1) + \frac{1}{2} \omega^2 x^2$
- (B) $H = \frac{1}{2} p \exp(2p) - \frac{1}{2} \omega^2 x^2$
- (C) $H = \frac{1}{2} \dot{x} \ln \dot{x} + \frac{1}{2} \omega^2 x^2$
- (D) $H = 2p \exp(2p-1) + \frac{1}{2} \omega^2 x^2$

83. If $f(x) = x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
 $= \pi - x$ for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

then the first Fourier coefficient a_0 is

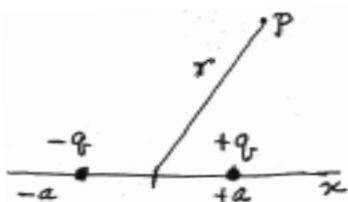
- (A) 2π
- (B) -2π
- (C) 1
- (D) 0

84. What is the speed of a particle having momentum $p = 5 \frac{\text{MeV}}{c}$ and total relativistic energy $E = 10 \text{ MeV}$?

- (A) $0.75 c$
- (B) $\frac{c}{\sqrt{3}}$
- (C) $\frac{c}{2}$
- (D) $\frac{c}{4}$

85. Two charges $+q$ and $-q$ are placed on the x -axis at $x = +a$ and $x = -a$ respectively. The electrostatic potential at a distance r from the origin will have terms proportional to

- (A) $\frac{1}{r}$ and $\frac{1}{r^2}$
- (B) $\frac{1}{r^2}$ only
- (C) $\frac{1}{r^2}, \frac{1}{r^3}$, and all higher powers
- (D) $\frac{1}{r^2}, \frac{1}{r^4}$, and all higher even powers



86. Consider a homonuclear diatomic molecule containing $N(N \geq 1)$ electrons, where the internuclear line of the molecule is lying along the z -axis of the coordinate system. If the position vectors of electrons are $\vec{r}_i (i = 1, \dots, N)$, which one of the following is correct?

- (A) Rotation about the z -axis is not a symmetry of the molecule.
- (B) Reflection of electrons in the xz plane and the transformations $\vec{r}_i \rightarrow -\vec{r}_i$ are the symmetries of the molecule.
- (C) Reflection of electrons in the xz plane is a symmetry but the transformations $\vec{r}_i \rightarrow -\vec{r}_i$ is not a symmetry of the molecule.
- (D) Neither reflection of electrons in xz plane nor the transformations $\vec{r}_i \rightarrow -\vec{r}_i$ is a symmetry of the molecule.

87. According to Debye theory, the heat capacity of a solid at low temperature is proportional to

- (A) T^6
- (B) T^3
- (C) $1/T$
- (D) T^3

88. Density of atoms in the plane (110) of a sc lattice will be

- (A) $\frac{1}{\sqrt{3}a^2}$
- (B) $\frac{1}{\sqrt{2}a^2}$
- (C) $\frac{\sqrt{2}}{a^2}$
- (D) $\frac{\sqrt{3}}{a^2}$

89. A tuned collector transistorized oscillator with a capacitor of 500 pF in the tank circuit oscillates at a frequency of 1 MHz . In order that the circuit can oscillate at 2 MHz , the inductor in the tank circuit should be

- (A) increased by a factor of 2
- (B) reduced by a factor of 2
- (C) reduced by a factor of 8
- (D) reduced by a factor of 4

90. According to the quark model

- (A) nucleons are made up of a pair of quarks.
- (B) mesons are made up of a pair of quarks.
- (C) mesons are made of two quarks and an antiquark.
- (D) mesons are made of a quark and an antiquark.

91. In a cathode ray oscilloscope an electron with velocity v_{0x} along x -axis enters in y -deflection plates across which a uniform electric field E_y is acting. The displacement of the electron in y -direction at time t after entering the field is

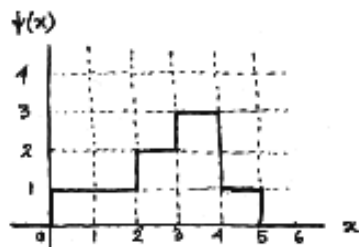
- (A) $-\frac{eE_y x^2}{2m v_{0x}^2}$
 (B) $\frac{-eE_y x^2}{2m v_{0x}}$
 (C) $\frac{-eE_y x}{2m v_{0x}}$
 (D) $\frac{-eE_y x}{2m v_{0x}^2}$

92. A particle is moving in one dimension under the potential $V(x) = \lambda(x^2 - \alpha x^4)$. At $t = 0$ the particle is released from a distance ' a ' from the origin. The motion will be bounded if

- (A) $|a| \geq \frac{4}{\sqrt{\alpha}}$
 (B) $|a| < \frac{1}{\sqrt{2\alpha}}$
 (C) $|a| \geq \frac{1}{\sqrt{3\alpha}}$
 (D) $|a| \leq \frac{4}{\sqrt{\alpha}}$

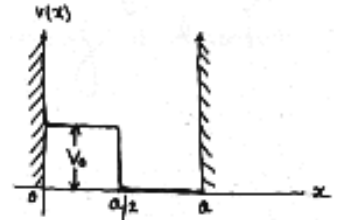
93. The profile of a wavefunction is given below ($\psi(x) = 0$ for $x < 0$ and also, for $x > 5$). What is the probability of finding the particle between $x = 2$ and $x = 4$?

- (A) $\frac{17}{64}$
 (B) $\frac{25}{64}$
 (C) $\frac{5}{8}$
 (D) $\frac{13}{16}$



94. A constant perturbation is given to the floor of an infinite square well, in the form of lifting the floor from $x = 0$ to $x = \frac{a}{2}$, as shown below. The first order correction to the n th energy level is given by

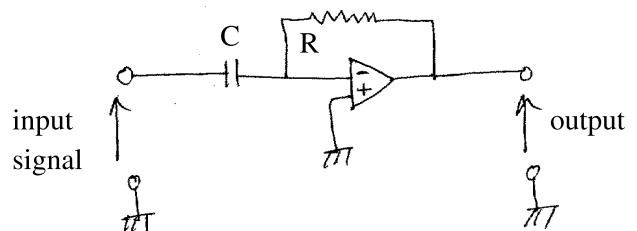
- (A) $\frac{V_0}{2}$
 (B) V_0
 (C) $2V_0$
 (D) $\frac{3}{2}V_0$



95. Given the mass of muon is 207 times that of an electron, the ionization potential of a muonic atom ($p^+ \mu^-$) is of the order of

- (A) 2.5 eV
 (B) 2.5 keV
 (C) 2.5 MeV
 (D) 0.025 eV

96. The following circuit is that of



- (A) A differentiator
 (B) An integrator
 (C) Low pass filter
 (D) A low frequency oscillator

97. A one dimensional harmonic oscillator of frequency ω with mass m and charge q is placed in a uniform electric field E . The energy levels are given by

- (A) $\left(n + \frac{1}{2}\right)\hbar\omega$
 (B) $\left(n + \frac{1}{2}\right)\hbar\omega - \frac{q^2 E^2}{2m\omega^2}$
 (C) $\left(n + \frac{1}{2}\right)\hbar\omega + \frac{q^2 E^2}{2m\omega^2}$
 (D) $\left(n + \frac{1}{2}\right)\hbar\omega + n \frac{q^2 E^2}{m\omega^2}$

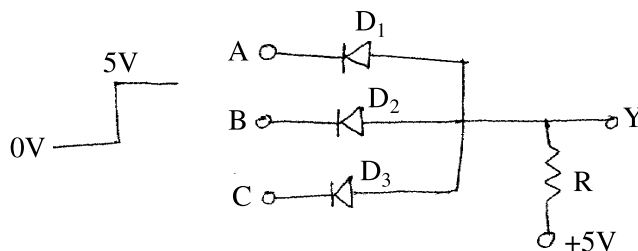
98. The Hamiltonian operator in certain orthonormalized basis in a three dimensional vector

space is written as $H = \begin{pmatrix} \epsilon & \eta & 0 \\ \eta & \epsilon & \eta \\ 0 & \eta & \epsilon \end{pmatrix}$ where both ϵ and η

have the dimension of energy. The results of measurement of energy of the system will be

- (A) $\epsilon, 0$ and $-\epsilon$
 (B) $\frac{\epsilon\eta}{\epsilon \pm \eta}$ and 0
 (C) ϵ and $\epsilon \pm \sqrt{2}\eta$
 (D) zero

99. The following circuit with D_1, D_2, D_3 as identical diodes in positive diode logic is that of



- (A) an OR gate
 (B) an AND gate
 (C) an EX-OR gate
 (D) a NAND gate

100. The number of lines observed in the Zeeman splitting of D-line of sodium is

- (A) Four
 (B) Six
 (C) Eight
 (D) Ten

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Y-18

ROUGH WORK

ROUGH WORK

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Y-20

ROUGH WORK